



B3 3-2 十字交乘法



概念 ① 乘積展開與因式分解



乘積展開

$$\begin{array}{r} x + 2 \\ \times) x + 3 \\ \hline x^2 \quad \square x \\ \square x \quad \square \\ \hline x^2 + \square x + \square \end{array}$$

$$(x+2)(x+3)=$$

因式分解

$$\begin{array}{r} x + \square \\ \times) x + \square \\ \hline x^2 \quad \square x \\ \square x \quad \square \\ \hline x^2 + 5x + 6 \end{array}$$

$$x^2+5x+6=$$

☆筆記

$$x^2+5x+6=(x+2)(x+3)$$

如何從 x^2+5x+6 找到 2 和 3 這兩個數字呢？



💡發現：



牛刀小試 ①

1. 請找出符合題意的二數

$$(1) \begin{cases} \text{相乘}=6 & \square \times \triangle = 6 \\ \text{相加}=7 & \square + \triangle = 7 \end{cases}$$

$$(2) \begin{cases} \text{相乘}=8 & \square \times \triangle = 8 \\ \text{相加}=9 & \square + \triangle = 9 \end{cases}$$

$$(3) \begin{cases} \text{相乘}=8 & \square \times \triangle = 8 \\ \text{相加}=6 & \square + \triangle = 6 \end{cases}$$

$$(4) \begin{cases} \text{相乘}=10 & \square \times \triangle = 10 \\ \text{相加}=7 & \square + \triangle = 7 \end{cases}$$

2. 負×負=正如： $(-2) \times (-3) = 6$

$$(1) \begin{cases} \text{相乘}=4 & \square \times \triangle = 4 \\ \text{相加}=-5 & \square + \triangle = -5 \end{cases}$$

$$(2) \begin{cases} \text{相乘}=12 & \square \times \triangle = 12 \\ \text{相加}=-7 & \square + \triangle = -7 \end{cases}$$

$$(3) \begin{cases} \text{相乘}=-12 & \square \times \triangle = -12 \\ \text{相加}=1 & \square + \triangle = 1 \end{cases}$$

$$(4) \begin{cases} \text{相乘}=-15 & \square \times \triangle = -15 \\ \text{相加}=-2 & \square + \triangle = -2 \end{cases}$$



因式分解 $x^2 + 5x + 6 =$ _____

$$\begin{array}{c} x \quad \times \quad \square \\ x \quad \times \quad \square \end{array} \begin{array}{c} \square \\ \square \end{array}$$

☆筆記

十字交乘法的步驟

$$x^2 + 5x + 6$$



☆利用上面的方法把 $x^2 + 5x + 6$ 因式分解為 $(x+2)(x+3)$ 的作法稱為_____。

〈練習〉因式分解 $x^2 + 7x + 12$



牛刀小試 2

因式分解下列各式

(1) $x^2 + 7x + 10$

$$\begin{array}{c} x \quad \times \quad \square \\ x \quad \times \quad \square \end{array} \begin{array}{c} \square \\ \square \end{array} \rightarrow 10$$

$$\square x + \square x = 7x$$

$$x^2 + 7x + 10 = (\quad)(\quad)$$

(2) $x^2 + 5x + 4$

$$\begin{array}{c} x \quad \times \quad \square \\ x \quad \times \quad \square \end{array} \begin{array}{c} \square \\ \square \end{array} \rightarrow 4$$

$$\square x + \square x = 5x$$

$$x^2 + 5x + 4 = (\quad)(\quad)$$

(3) $x^2 + 8x + 15$

$$\begin{array}{c} x \quad \times \quad \square \\ x \quad \times \quad \square \end{array} \begin{array}{c} \square \\ \square \end{array}$$

$$\square x + \square x = 8x$$

$$x^2 + 8x + 15 = (\quad)(\quad)$$

(4) $x^2 + 6x + 8$

$$\begin{array}{c} x \quad \times \quad \square \\ x \quad \times \quad \square \end{array} \begin{array}{c} \square \\ \square \end{array}$$

$$\square x + \square x = 6x$$

$$x^2 + 6x + 8 = (\quad)(\quad)$$

(5) $x^2 + 11x + 18$

$$\begin{array}{c} x \quad \times \quad \square \\ x \quad \times \quad \square \end{array} \begin{array}{c} \square \\ \square \end{array}$$

$$\square x + \square x = 11x$$

$$x^2 + 9x + 18 = (\quad)(\quad)$$

(6) $x^2 + 9x + 20$

$$\begin{array}{c} x \quad \times \quad \square \\ x \quad \times \quad \square \end{array} \begin{array}{c} \square \\ \square \end{array}$$

$$\square x + \square x = 9x$$

$$x^2 + 9x + 20 = (\quad)(\quad)$$

(7) $x^2 + 12x + 20$

$$\begin{array}{c} x \quad \times \quad \square \\ x \quad \times \quad \square \end{array} \begin{array}{c} \square \\ \square \end{array}$$

$$\square x + \square x = 12x$$

$$x^2 + 12x + 20 = (\quad)(\quad)$$

(8) $x^2 + 9x + 14$

$$\begin{array}{c} x \quad \times \quad \square \\ x \quad \times \quad \square \end{array} \begin{array}{c} \square \\ \square \end{array}$$

$$\square x + \square x = 9x$$

$$x^2 + 9x + 14 = (\quad)(\quad)$$



例題 ① 十字交乘法練習 1



利用十字交乘法做因式分解

(1) $x^2 + 6x + 5$

(2) $x^2 + 6x + 8$

☆筆記



牛刀小試 3

因式分解下列各式

(1) $x^2 + 10x + 21$

(2) $x^2 + 10x + 24$

(3) $x^2 + 14x + 24$

(4) $x^2 + 10x + 16$

(5) $x^2 + 21x + 20$

(6) $x^2 + 13x + 30$

(7) $x^2 + 11x + 30$

(8) $x^2 + 12x + 35$

**例題****②****十字交乘法練習 2**

利用十字交乘法做因式分解

(1) $x^2 + 7x + 6$

(2) $x^2 - 5x + 6$

☆筆記

**牛刀小試 4****1. 因式分解下列各式**

(1) $x^2 + 17x + 16$

(2) $x^2 + 23x + 22$

(3) $x^2 + 10x + 24$

(4) $x^2 + 13x + 36$

2. 因式分解下列各式

(1) $x^2 - 5x + 4$

(2) $x^2 - 16x + 15$

(3) $x^2 - 9x + 14$

(4) $x^2 - 10x + 21$

**例題****③ 十字交乘法練習 3**

利用十字交乘法做因式分解

(1) $x^2 + x - 12$

(2) $x^2 - 11x - 12$

☆筆記

**牛刀小試 5****1. 因式分解下列各式**

(1) $x^2 + x - 6$

(2) $x^2 + 5x - 6$

(3) $x^2 + 9x - 10$

(4) $x^2 + 3x - 10$

2. 因式分解下列各式

(1) $x^2 - x - 6$

(2) $x^2 - 5x - 6$

(3) $x^2 - 4x - 21$

(4) $x^2 - 20x - 21$



乘積展開

$$\begin{array}{r}
 2x + 5 \\
 \times) 3x + 4 \\
 \hline
 \square x^2 \quad \square x \\
 \quad \square x \quad \square \\
 \hline
 \square x^2 + \square x + \square
 \end{array}$$

$$(2x+5)(3x+4) = \underline{\hspace{2cm}} \quad 6x^2 + 23x + 20 = \underline{\hspace{2cm}}$$

因式分解

$$\begin{array}{r}
 \square x \quad \square \\
 \times) \square x \quad \square \\
 \hline
 \square x^2 \quad \square x \\
 \quad \square x \quad \square \\
 \hline
 6x^2 + 23x + 20
 \end{array}$$

☆筆記

☆利用十字交乘法因式分解 $6x^2 + 23x + 20$ 

牛刀小試 6

因式分解下列各題

(1) $2x^2 + 15x + 7$

$$\begin{array}{r}
 x \quad \times \quad \square \\
 2x \quad \times \quad \square \\
 \hline
 \square x + \square x = 15x
 \end{array}$$

$$2x^2 + 15x + 7 = (\underline{\hspace{2cm}})(\underline{\hspace{2cm}})$$

(2) $3x^2 + 8x + 5$

$$\begin{array}{r}
 x \quad \times \quad \square \\
 3x \quad \times \quad \square \\
 \hline
 \square x + \square x = 8x
 \end{array}$$

$$3x^2 + 8x + 5 = (\underline{\hspace{2cm}})(\underline{\hspace{2cm}})$$

(3) $2x^2 + 7x + 5$

$$\begin{array}{r}
 x \quad \times \quad \square \\
 2x \quad \times \quad \square \\
 \hline
 \square x + \square x = 7x
 \end{array}$$

$$2x^2 + 7x + 5 = (\underline{\hspace{2cm}})(\underline{\hspace{2cm}})$$

(4) $6x^2 + 25x + 24$

$$\begin{array}{r}
 2x \quad \times \quad \square \\
 3x \quad \times \quad \square \\
 \hline
 \square x + \square x = 25x
 \end{array}$$

$$6x^2 + 25x + 24 = (\underline{\hspace{2cm}})(\underline{\hspace{2cm}})$$

(5) $6x^2 + 31x + 18$

$$\begin{array}{r}
 2x \quad \times \quad \square \\
 3x \quad \times \quad \square \\
 \hline
 \square x + \square x = 31x
 \end{array}$$

$$6x^2 + 31x + 18 = (\underline{\hspace{2cm}})(\underline{\hspace{2cm}})$$

(6) $6x^2 + 37x + 6$

$$\begin{array}{r}
 x \quad \times \quad \square \\
 6x \quad \times \quad \square \\
 \hline
 \square x + \square x = 37x
 \end{array}$$

$$6x^2 + 37x + 6 = (\underline{\hspace{2cm}})(\underline{\hspace{2cm}})$$

**例題****4****十字交乘法練習 4**

利用十字交乘法做因式分解

(1) $6x^2 + 17x + 12$

(2) $2x^2 - x - 36$

☆筆記

**牛刀小試 7**

1. 因式分解下列各式

(1) $10x^2 + 19x + 6$

(2) $12x^2 + 23x + 10$

(3) $14x^2 - 29x + 12$

(4) $16x^2 - 40x + 21$

2. 因式分解下列各式

(1) $2x^2 + x - 21$

(2) $3x^2 + x - 10$

(3) $12x^2 - 5x - 2$

(4) $12x^2 - 8x - 15$



例題 ⑤ 十字交乘法練習 5



若多項式 $5x^2 + 17x - 12$ 可因式分解成 $(x+a)(bx+c)$ ，其中 a 、 b 、 c 均為整數，則 $a+c$ 之值為何？

【108 會考第 8 題】

☆筆記



牛刀小試 8

1. 若多項式 $7x^2 + 11x - 6$ 可因式分解成 $(x+a)(bx+c)$ 其中 a 、 b 、 c 均為整數，則 $a+c$ 之值為何？
2. 若多項式 $5x^2 + 18x - 8$ 可因式分解成 $(x+a)(bx+c)$ 其中 a 、 b 、 c 均為整數，則 $a+c$ 之值為何？
3. 若多項式 $7x^2 - 3x - 10$ 可因式分解成 $(x+a)(bx+c)$ 其中 a 、 b 、 c 均為整數，則 $a+c$ 之值為何？
4. 若多項式 $6x^2 - 5x - 14$ 可因式分解成 $(x+a)(bx+c)$ 其中 a 、 b 、 c 均為整數，則 $a+c$ 之值為何？



牛刀小試 1

- (1) 1, 6
- (2) 1, 8
- (3) 2, 4
- (4) 2, 5
- (5) (-1), (-4)
- (6) (-3), (-4)
- (7) (-3), 4
- (8) (-5), 3

牛刀小試 2

1.

$$(1) \begin{array}{r} x^2 + 7x + 10 \\ x \quad \times \quad +2 \\ x \quad \times \quad +5 \\ \hline 2x + 5x = 7x \end{array}$$

$$x^2 + 7x + 10 = (x+2)(x+5)$$

$$(2) \begin{array}{r} x^2 + 5x + 4 \\ x \quad \times \quad +1 \\ x \quad \times \quad +4 \\ \hline 1x + 4x = 5x \end{array}$$

$$x^2 + 5x + 4 = (x+1)(x+4)$$

$$(3) \begin{array}{r} x^2 + 8x + 15 \\ x \quad \times \quad +3 \\ x \quad \times \quad +5 \\ \hline 3x + 5x = 8x \end{array}$$

$$x^2 + 8x + 15 = (x+3)(x+5)$$

$$(4) \begin{array}{r} x^2 + 6x + 8 \\ x \quad \times \quad +2 \\ x \quad \times \quad +4 \\ \hline 2x + 4x = 6x \end{array}$$

$$x^2 + 6x + 8 = (x+2)(x+4)$$

2.

$$(1) \begin{array}{r} x^2 + 11x + 18 \\ x \quad \times \quad +2 \\ x \quad \times \quad +9 \\ \hline 2x + 9x = 11x \end{array}$$

$$x^2 + 11x + 18 = (x+2)(x+9)$$

$$(2) \begin{array}{r} x^2 + 9x + 20 \\ x \quad \times \quad +4 \\ x \quad \times \quad +5 \\ \hline 4x + 5x = 9x \end{array}$$

$$x^2 + 9x + 20 = (x+4)(x+5)$$

$$(3) \begin{array}{r} x^2 + 12x + 20 \\ x \quad \times \quad +2 \\ x \quad \times \quad +10 \\ \hline 2x + 10x = 12x \end{array}$$

$$x^2 + 12x + 20 = (x+2)(x+10)$$

$$(4) \begin{array}{r} x^2 + 9x + 14 \\ x \quad \times \quad +2 \\ x \quad \times \quad +7 \\ \hline 2x + 7x = 9x \end{array}$$

$$x^2 + 9x + 14 = (x+2)(x+7)$$

牛刀小試 3

- (1) $(x+3)(x+7)$
- (2) $(x+4)(x+6)$
- (3) $(x+2)(x+12)$
- (4) $(x+2)(x+8)$
- (5) $(x+1)(x+20)$
- (6) $(x+3)(x+10)$
- (7) $(x+5)(x+6)$
- (8) $(x+5)(x+7)$

牛刀小試 4

1.

- (1) $(x+1)(x+16)$
- (2) $(x+1)(x+22)$
- (3) $(x+4)(x+6)$
- (4) $(x+4)(x+9)$

2.

- (1) $(x-1)(x-4)$
- (2) $(x-1)(x-15)$
- (3) $(x-2)(x-7)$
- (4) $(x-3)(x-7)$

牛刀小試 5

1.

- (1) $(x+3)(x-2)$
- (2) $(x+6)(x-1)$
- (3) $(x+10)(x-1)$
- (4) $(x+5)(x-2)$

2.

- (1) $(x+2)(x-3)$
- (2) $(x-6)(x+1)$
- (3) $(x+3)(x-7)$
- (4) $(x+1)(x-21)$

牛刀小試 6

$$(1) \begin{array}{r} 2x^2 + 17x + 12 \\ 1x \quad \times \quad +7 \\ 2x \quad \times \quad +1 \\ \hline 14x + 1x = 15x \end{array}$$

$$2x^2 + 17x + 12 = (x+7)(2x+1)$$

$$(2) \begin{array}{r} 3x^2 + 8x + 5 \\ 1x \quad \times \quad +1 \\ 3x \quad \times \quad +5 \\ \hline 3x + 5x = 8x \end{array}$$

$$3x^2 + 8x + 5 = (x+1)(3x+5)$$

$$(3) \begin{array}{r} 2x^2 + 7x + 5 \\ 1x \quad \times \quad +1 \\ 2x \quad \times \quad +5 \\ \hline 2x + 5x = 7x \end{array}$$

$$2x^2 + 7x + 5 = (x+1)(2x+5)$$

$$(4) \begin{array}{r} 6x^2 + 25x + 24 \\ 2x \quad \times \quad +3 \\ 3x \quad \times \quad +8 \\ \hline 9x + 16x = 25x \end{array}$$

$$6x^2 + 25x + 24 = (2x+3)(3x+8)$$

$$(5) \begin{array}{r} 6x^2 + 31x + 18 \\ 2x \quad \times \quad +9 \\ 3x \quad \times \quad +2 \\ \hline 27x + 4x = 31x \end{array}$$

$$6x^2 + 31x + 18 = (2x+9)(3x+2)$$

$$(6) \begin{array}{r} 6x^2 + 37x + 6 \\ 1x \quad \times \quad +6 \\ 6x \quad \times \quad +1 \\ \hline 36x + 1x = 37x \end{array}$$

$$6x^2 + 37x + 6 = (x+6)(6x+1)$$

牛刀小試 7

1.

- (1) $(5x+2)(2x+3)$
- (2) $(4x+5)(3x+2)$
- (3) $(7x-4)(2x-3)$
- (4) $(4x-3)(4x-7)$

2.

- (1) $(x-3)(2x+7)$
- (2) $(x+2)(3x-5)$
- (3) $(3x-2)(4x+1)$
- (4) $(2x-3)(6x+5)$

牛刀小試 8

1. -1

2. 2

3. -9

4. 5